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ABOUT THE PROPERTIES OF NETWORKS

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ANNOTATION

In this paper, we investigate the following (1) the product of cs-networks, the product of cs*-networks is cs*-networks, the image of cs-network by sequence-covering map is cs-network, the image of cs*-network by 1-sequence-covering map is cs*-network, the product of k-networks is a k-network, the image of k-network by compact-covering map is a k-network.

Keywords: cs-network, cs*-network, k-network, sequence-covering map, compact-covering map.

НЕКОТОРЫЕ СВОЙСТВА СЕТЕЙ

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АННОТАЦИЯ

В этой статье мы исследуем следующее: произведение cs-сетей, произведение cs*-сетей - это cs*-сети, образ cs-сети с помощью карты последовательного покрытия - cs-сеть, образ cs-сети cs* -сеть с помощью карты покрытия с 1-последовательностью - это cs* -сеть, произведение k-сетей - это k-сеть, образ k-сети с помощью карты компактного покрытия - это k-сеть.

Ключевые слова: cs-сеть, cs*-сеть, k-сеть, последовательное покрытие, компактное покрытие.

1. Introduction

To determine preserving topological properties of topological spaces by product and continuous map is one of the central question of general topology. The networks (cs, cs*, k) are characterized by important properties of topological spaces. Some properties of networks (cs, cs*, k) and of covering maps (sequence, 1-sequence, compact) are discussed in [1, 3-12].

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2. Main Results

Let X be a T_1 topological space and $P = \{P_\alpha : P \subset X\}$ be a family with $x \in \cap P_\alpha$.

Definition 2.1. A sequence $\{x_n\}$ in X is called eventually in P if $\{x_n\}$ converges to x, and there exists $m \epsilon N$ such that $\{x\} \cup \{x_n : n \geq m\} \subset P$.

Definition 2.2. The family P is called a network at point $x \in X$ if for each neighborhood of x there exists $P \in P$ such that $P \in U$.

Definition 2.3. The family P is called a network at point $x \in X$ if for any sequence $\{x_n\}$ converging to x and a neighborhood U of x, there exists $P \in P$such that $P \subset U$ and $\{x_n\}$ is eventually in P.

Definition 2.4. The family P is called a cs*-network at a point $x \in X$ if whenever $\{x_n\}$ is a sequence converging to a point $x \in U$ with U open is X, then $\{x_{n_i} : i \in N\} \subset P \subset U$ for some subsequence $\{x_{n_i}\}$ of $\{x_n\}$ and some $P \in P$.

Proposition 2.5. If the families P and T are cs-networks respectively at points $x \in X$ and $y \in Y$, then the family $P \times T$ is cs-network too at point $(x, y) \in X \times Y$.

Prof. Let G be a neighborhood of point (x, y) and $\{x_n\}$, $\{y_n\}$ are some sequences converging to points x and y respectively. It is easy to see that there exist neighborhoods U, V of points x and y respectively, such that $U \times V \subset G$. Moreover, there exist $P \in P$, $T \in T$ and $n_0 \in N$, $m_0 \in N$ that $\{x_n\} \subset P \subset U$ and $\{y_k\} \subset T \subset V$ for each $n > n_0$, $k > m_0$. We take $m = \max(n_0, m_0)$, then $\{(x_n, y_n)\} \subset P \times T \subset G$ for each $n > m$. Hence, $P \times T$ is cs-network at point (x, y).

Corollary 2.6. The families $P_i, i = \overline{1, n}$ are cs-networks at points $x_i \in X_i$ respectively, then their product $\prod_{i=1}^{n} P_i$ is a cs-network too at point $(x_1, x_2, \ldots, x_n) \in \prod_{i=1}^{n} X_i$.

Example 2.7. Let $X = [= 3, 3]$ be space. It is easy to see the family $P = \{\cup \left(1 - \frac{1}{n}, 1 + \frac{1}{n}\right)\}$ is a cs-network at point $x = 1$ and $T = \{\cup \left(2 - \frac{1}{n}, 2 + \frac{1}{n}\right)\}$ is a cs-network at point $y = 2$, where $n \in N$. For each neighborhood G of point $A(x, y)$ we take $r = min_{B \in \partial G}\{d(A, B)\}$, where d is metric in X. Next we take $U = \left(1 - \frac{r}{3}, 1 + \frac{r}{3}\right)$, $\left(V = (2 - \frac{r}{3}, 2 + \frac{r}{3}\right)$, then $U \times V \subset G$. We can find $n_0 \in N$ such that for $P = \left(1 - \frac{1}{n_0}, 1 + \frac{1}{n_0}\right)$, $T = \left(2 - \frac{1}{n_0}, 2 + \frac{1}{n_0}\right)$ this attitude $P \times T \subset U \times V \subset G$ is understandable. Therefore, $P \times T$ is a cs-network too.

Proposition 2.8. If the families P and T are cs*-networks respectively at points $x \in X, y \in Y$ then a family $P \times T$ is cs*-network too at point $(x, y) \in X \times Y$.

Proof. In this case again let G be a neighborhood of point (x, y) and $\{x_n\}$ and $\{y_n\}$ are some sequences converging to points x and y respectively and is known there exists neighborhoods U, V of points x and y respectively, such that $U \times V \subset G$. Moreover, by definition of cs*-network there exist $P \in P, T \in T$ and subsequences $\{x_{n_i} : i \in N\}$ and $\{y_{n_j} : j \in N\}$ of sequences $\{x_n\}$ and $\{y_n\}$ respectively, such that $\{x_{n_i} : i \in N\} \subset P \subset U$ and $\{y_{n_j} : j \in N\} \subset T \subset V$. Afterward we re-numbered subsequences and we have $\{(x_{n_k}, y_{n_k}) : k \in N\} \subset P \times T \subset G$.

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Hence, $P \times T$ is a cs*-network at the point (x, y) and we have proved the proposition 2.8.
Corollary 2.9. The families $P_i, i = \overline{1, n}$ are cs*-networks respectively at points $x_i \in X_i$, then their product $\prod_{i=1}^{n} P_i$ is a cs*-network at the point $(x_1, x_2, \dots, x_n) \in \prod_{i=1}^{n} X_i$.
Definition 2.10. [8]. Let $f: X \to Y$ be a map continuous and onto

1) f is a sequence-covering map if each convergent sequence (includes its limit point) of Y is the image of some convergent sequence of X.
2) f is a 1-sequence-covering map if for each $y \in Y$, there is $x \in f^{-1}(y)$ such that whenever $\{y_n\}$ is a sequence converging to y in Y there is sequence $\{x_n\}$ converging to x in X with each $x_n \in f^{-1}(y_n)$.

Remark 2.11. 1-sequence-covering map ⇒ sequence-covering map.
Proposition 2.12. If $f: X \to Y$ is sequence-covering map and P is a cs-network at point $x_0 \in X$, then $f(P) = \{f(P): P \in P\}$ is a cs-network at the point $y_0 = f(x_0)$.
Proof. By definition of continuous map for each neighborhood V of point y_0 there exists a neighborhood U of points x_0 such that $f(U) \subset V$. Since the family P is cs-network at the point x_0, there exists $P \in P$ such that $P \subset U$. Therefore, there exists $T = f(P) \in f(P)$ such that $T \subset V$. Now we will show that for each sequence $\{y_n\}$ converging to y_0 there is $m \in N$ such that $\{y_n\} \subset T$ for every $n > m$. We have that f is sequence-covering map, so the sequence $\{y_n\}$ is the image of some sequence $\{x_n\}$ of X converging to x_0. Then there exists $m \in N$ such that $\{x_n\} \subset P$ for every $n > m$, so $\{f(x_n)\} = \{y_n\} \subset f(P) = T$ for every $n > m$. So $f(P)$ is cs-network at point y_0.
Proposition 2.13. If $f: X \to Y$ 1-sequence covering map and P is a cs*-network at point $x_0 \in X$, then $f(P) = \{f(P): P \in P\}$ is cs*-network at point $y_0 = f(x_0)$.
Proof. Us sufficient show that for every sequence $\{y_n\}$ converging to point $y_0 \in V$ with V open in Y there exists subsequence $\{y_{n_i}: i \in N\}$ and $T \in f(P)$ such that $\{y_{n_i}: i \in N\} \subset T \subset V$. We have that f is 1-sequence covering map. Therefore, there exist $z_0 \in f^{-1}(y_0)$ and $x_n \in f^{-1}(y_n)$ such that $\{x_n\}$ is a converging sequence to z_0. In addition, P is a cs*-network at a point x_0, so there exists subsequence $\{x_{n_i}: i \in N\}$ of $\{x_n\}$ and $P \in P$ such that $\{x_{n_i}\} \subset P$, therefore, $\{f(x_{n_i}) = y_{n_i}\} \subset \{y_n\} \subset f(P) = T$. Hence, $f(P) = \{f(P): P \in P\}$ is cs*-network at the point y_0.
Definition 2.14. P is called k-network if whenever $K \subset U$ with K compact and U open in X, then $K \subset \bigcup P' \subset U$ for some finite $P' \subset P$.
Let $f: X \to Y$ be a map continuous and onto.
Definition 2.15. The map f is called *compact-covering map* if each compact subset of Y is the image of some compact subset of X.
Definition 2.16. If the families P and T are k-networks respectively in X and Y, then the family $P \times T$ is k-network in $X \times Y$.
Proof. Let K be a compact subset of $X \times Y$ and $K \subset U$ with U open in $X \times Y$. We denote by K_1 and K_2 the projects of K to X and Y respectively. It is easy to see K_1 and K_2 are compact subsets of X and Y respectively. Let be $K_1 \subset U_1$ and $K_2 \subset U_2$, for some open subsets U_1, U_2. We have that P and T are k-networks. So there exist finite subfamilies $P' = \{P_i: P_i \in P, i = \overline{1, n}\}$ and $T' = \{T_j: T_j \in T, j = \overline{1, m}\}$ of P

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and T respectively such that $K_1 \subset \{\bigcup_{i=1}^{n} P_i\} \subset U_1$ and $K_2 \subset \{\bigcup_{j=1}^{m} T_j\} \subset U_2$. Then it is easy to see $K \subset (K_1 \times K_2) \cap U \subset (\{\bigcup_{i=1}^{n} P_i\} \times \{\bigcup_{j=1}^{m} T_j\}) \cap U \subset (U_1 \times U_2) \cap U \subset U$.
As you know, $\{\bigcup_{i=1}^{n} P_i\} \times \{\bigcup_{j=1}^{m} T_j\} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{m} P_i \times T_j$, where $P_i \times T_j \in P \times T$. Hence, $P \times T$ is k-network in $X \times Y$ too.
Corollary 2.17. The families $P_i, i = \overline{1,n}$ are k-networks respectively in X_i then their product $\prod_{i=1}^{n} P_i$ is k-network in $\prod_{i=1}^{n} X_i$.
Proposition 2.18. If $f: X \to Y$ is compact-covering map and P is a k-network in X, then $f(P) = \{f(P): P \in P\}$ is k-network in Y.
Proof. Let be F is compact and V is open with $F \subset V$. By definition of compact-covering map there exists compact subset K of X such that $f(K) = F$. We have that f is continuous map so $f^{-1}(V)$ is open in X and $K \subset f^{-1}(V)$. Otherwise, P is a k-network so there exists finite $P' \subset P$ such that $K \subset \bigcup P' \subset f^{-1}(V)$. Thus implies $F \subset f(\bigcup P') = \bigcup f(P') \subset V$. Therefore, $f(P)$ is k-network in Y.

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