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SOME METHODOLOGICAL ADVICE FOR CALCULATION OF PARTIAL DERIVATIVES OF FUNCTIONS OF MULTIPLE VARIABLES

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Annotation:
In this paper provided a number of theoretical and logical foundations, which is impossible to correctly calculate the partial derivatives of functions of many variables. Typical variants of a function of many variables are given, as well as methodological recommendations for calculating partial derivatives using a formula and definition of problem. Solutions to numerous problems are shown accompanied by useful methodological advice that allows you to correctly calculate derivatives and differentials. A minimum of theoretical knowledge is given, which is necessary to calculate the partial derivatives of specific functions, as well as tasks and exercises for independent work.

Keywords: increment of a function, continuity at a point, derivative rules, expression, definition of a partial derivative, equality, limit, continuity of partial derivatives, tangent plane equation, paraboloid, argument.

Practice shows that not all students understand the theoretical, logical foundations, without which it is impossible to correctly calculate partial derivatives by formula and by definition. Some people know the theoretical foundations of the position, but they know them formally.

When solving problems, everyone must know (possess) the minimum theoretical knowledge that is necessary to compute a partial derivative for a particular function [1, p.191].

In order to confirm the above remarks, on the basis of several functions given in this work, a test of the environment of university students was conducted. According to the results, if we draw conclusions, both of them made some mistakes. The results of right and wrong are 45 to 55 percent. This, of course, is not a good result.

In this article, we will present typical functions that students can apply this knowledge to solving other problems when further studying partial differential computing.

Example 1. Prove that the

$$u=\begin{cases}\frac{x^3+y^3}{x^2+y^2}, x^2+y^2\neq 0,\\ 0, \quad x^2+y^2=0,\end{cases}$$

has partial derivatives at O(0,0), but is not differentiable at this point.

Proof. Since

$$u(x,0)=\begin{cases}x, x\neq 0,\\ 0, x=0,\end{cases}$$

i.e.

$$u(x,0)=x,$$

that

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$$\frac{\partial u}{\partial x}(0,0) = \frac{d}{dx}u(x,0)|_{x=0} = 1.$$

In the same way, we get

$$\frac{\partial u}{\partial y}(0,0) = 1.$$

And so the function has partial derivatives at point O(0,0).$u(x, y)$

Let us prove that the function is not differentiable at point O(0,0).$u(x, y)$

Suppose the opposite. Then the increment of the function at this point, equal

$$\Delta u = u(\Delta x, \Delta y) - u(0,0) = \frac{\Delta x^3 + \Delta y^3}{\Delta x^2 + \Delta y^2},$$

to can be represented as

$$\Delta u = \frac{\partial u}{\partial x}(0,0)\Delta x + \frac{\partial u}{\partial y}(0,0)\Delta y + o(p),$$

Where is. $p = \sqrt{\Delta x^2 + \Delta y^2}$

Since

$$\frac{\partial u}{\partial x}(0,0) = \frac{\partial u}{\partial y}(0,0) = 1.$$

From the condition of differentiability we derive

$$\frac{\Delta x^3 + \Delta y^3}{\Delta x^2 + \Delta y^2} = \Delta x + \Delta y + o(p),$$

either i.e

$$-\frac{\Delta x \Delta y^2 + \Delta x^2 \Delta y}{\Delta x^2 + \Delta y^2} = o\left(\sqrt{\Delta x^2 + \Delta y^2}\right),$$

.

$$\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{\Delta x \Delta y^2 + \Delta x^2 \Delta y}{(\Delta x^2 + \Delta y^2)^{3/2}} = 0.$$

Let us show that this limit does not actually exist.

Let them strive to zero in such a way that Δx и $\Delta y \Delta y = k\Delta x (k \neq 0)$.

Then we'll get

$$\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0 \\ (\Delta y = k\Delta x)}} \frac{\Delta x \Delta y^2 + \Delta x^2 \Delta y}{(\Delta x^2 + \Delta y^2)^{3/2}} = \lim_{\Delta x \to 0} \frac{\Delta x^3 (k^2 + k)}{\Delta x^3 (1 + k^2)^{3/2}} = \frac{k^2 + k}{(1 + k^2)^{3/2}}.$$

Since the magnitude takes different values for different , the specified limit does not exist. It follows that the assumption made is incorrect, and therefore the function is differentiable at point O(0,0).$\frac{k^2+k}{(1+k^2)^{3/2}} k u(x, y)$

Example 2. Prove that the

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$$u=\begin{cases}(x^2+y^2)\sin\dfrac{1}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0,\\ 0, & x^2+y^2=0,\end{cases}$$

has partial derivatives in the neighborhood of the O(0,0) point and is differentiable at the O, but the partial derivatives are not continuous at the O point.

Proof. At all points except the point O(0,0), partial derived functions can be found by calculating the derived functions according to the usual rules$u=(x,y)$

$$(x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}}.$$

For example

$$\frac{\partial u}{\partial x}(x,y)=2x\,sin\frac{1}{\sqrt{x^2+y^2}}+(x^2+y^2)\,cos\frac{1}{\sqrt{x^2+y^2}}\left(-\frac{1}{2}\right)(x^2+y^2)^{-3/2}2x$$
$$=2x\,sin\frac{1}{\sqrt{x^2+y^2}}-\frac{x}{\sqrt{x^2+y^2}}cos\frac{1}{\sqrt{x^2+y^2}}$$

at

$$x^2+y^2\neq 0.$$

At point O(0,0) this formula loses its meaning. However, this does not mean that it does not exist, since the expression for was obtained under the condition . To find it, let's use the definition of a partial derivative. Since $\frac{\partial u}{\partial x}(0,0)\frac{\partial u}{\partial x}(x,y)x^2+y^2\neq 0\frac{\partial u}{\partial x}(0,0)$

$$u(x,0)=\begin{cases}x^2\,sin\dfrac{1}{|x|}, & x\neq 0,\\ 0, & x=0,\end{cases}$$

that

$$\Delta_x u=u(\Delta x,0)-u(0,0)=\Delta x^2\,sin\frac{1}{|\Delta x|}.$$

Hence

$$\lim_{\Delta x\to 0}\frac{\Delta_x u}{\Delta x}=\lim_{\Delta x\to 0}\Delta x\sin\frac{1}{|\Delta x|}=0\,,$$

i.e. $\frac{\partial u}{\partial x}(0,0)=0.$

Similarly, it can be proved that

$$\frac{\partial u}{\partial y}(0,0)=0.$$

So, the function has partial derivatives in the neighborhood of the point O(0,0).$u(x,y)$

Let us prove that the function is differentiable at point O(0,0). $u(x,y)$

To do this, we need to prove that the increment of the function

$$\Delta u=u(\Delta x,\Delta y)-u(0,0)=(\Delta x^2+\Delta y^2)\sin\frac{1}{\sqrt{\Delta x^2+\Delta y^2}}$$

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can be represented as

$$\Delta u = \frac{\partial u}{\partial x}(0{,}0)\Delta x + \frac{\partial u}{\partial y}(0{,}0)\Delta y + o\left(\sqrt{\Delta x^2 + \Delta y^2}\right),$$

i.e., equality is fair (take into account that $\frac{\partial u}{\partial y}(0{,}0) = \frac{\partial u}{\partial y}(0{,}0) = 0$)

$$(\Delta x^2 + \Delta y^2)\sin\frac{1}{\sqrt{\Delta x^2 + \Delta y^2}} = o\left(\sqrt{\Delta x^2 + \Delta y^2}\right).$$

But this equality is obvious. As

$$\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{(\Delta x^2 + \Delta y^2)\, sin\frac{1}{\sqrt{\Delta x^2 + \Delta y^2}}}{\sqrt{\Delta x^2 + \Delta y^2}} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} (\Delta x^2 + \Delta y^2)\, sin\frac{1}{\sqrt{\Delta x^2 + \Delta y^2}} = 0.$$

Thus, the function is differentiable at point O(0,0).$u(x, y)$

Finally, let us prove that the partial derivative is not continuous at point O(0,0). $\frac{\partial u}{\partial x}(x, y)$

Obviously, the first term tends to zero at . $2x\, sin\frac{1}{\sqrt{x^2+y^2}} M(x, y) \to O(0{,}0)$

The second term

$$\left(-\frac{x}{\sqrt{x^2 + y^2}} cos\frac{1}{\sqrt{x^2 + y^2}}\right)$$

has no limit at . $M(x, y) \to O(0{,}0)$

In fact, if a point tends to the point O(0,0) along the ray , then on this ray the specified term is equal to$M(x, y) y = kx\ (k \neq 0, x > 0)$

$$-\frac{1}{\sqrt{1 + k^2}} cos\frac{1}{x\sqrt{1 + k^2}}$$

and obviously has no limit at . So, the limit is not continuous at point O(0,0).$x \to 0\, \frac{\partial u}{\partial y}(x, y)$

This typical example shows that partial differential continuity is only a sufficient, but not a necessary condition for the differentiability of a function.

Example 3. Make an equation for the tangent plane to the paraboloid at a point and find the normal to the paraboloid at that point.$u = x^2 + y^2 N_0(1{,}2{,}5)$

Decision. Let a point on the plane . Since$M_0(1{,}2) - Oxy$

$$\frac{\partial u}{\partial x} = 2x, \frac{\partial u}{\partial y} = 2y$$

that

$$\frac{\partial u}{\partial x}(M_0) = 2, \frac{\partial u}{\partial y}(M_0) = 4.$$

Taking into account also that and , we obtain the desired equation of the tangent plane$u(M_0) = 5$

$$2(x - 1) + 4(y - 2) - (u - 5) = 0,$$

or

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$$2x + 4y - u - 5 = 0.$$

The vector is normal to the paraboloid at . $\boldsymbol{n} = \{2,4,-1\} N_0$

Example 4. Find partial derived functions from arguments and .$u = f(x, xy, xyz) x, yz$

Decision. This function is a complex function of variables and :x, yz

$$u = f(x_1, x_2, x_3),$$

Where is. $x_1 = x, x_2 = xy, x_3 = xyz$

Let us denote the partial derivative of the function by the argument , through (the function depends on the same arguments as the function , i.e. . . $u = f(x_1, x_2, x_3) x_i f_i' (i = 1,2,3) f_i' f f_i' = f_i'(x, xy, xyz)$

Applying the formula [1, p. 191], we get

$$\frac{\partial u}{\partial x} = f_i' \cdot 1 + f_2' \cdot y + f_3' \cdot yz, \frac{\partial u}{\partial y} = f_2' \cdot x + f_3' \cdot xz, \ \frac{\partial u}{\partial z} = f_3' \cdot xy.$$

Example 5. Find Differential Function:

(a) At the point $u = e^{x^2} + y^2 + z^2 M(0,1,2)$;

b) at the point $u = f(x + y^2, y + x^2) M(-1,1)$.

Decision.

a) We have, $\frac{\partial u}{\partial x} = e^{x^2} + y^2 + z^2 \cdot 2x \qquad \frac{\partial u}{\partial x}(M) = 0.$

$$\frac{\partial u}{\partial y} = e^{x^2} + y^2 + z^2 \cdot 2y, \frac{\partial u}{\partial y}(M) = 2c^5;$$

$$\frac{\partial u}{\partial z} = e^{x^2} + y^2 + z^2 \cdot 2z, \qquad \frac{\partial u}{\partial z}(M) = 4e^5.$$

Therefore

$$du|_M = \frac{\partial u}{\partial x}(M)dx + \frac{\partial u}{\partial y}(M)dy + \frac{\partial u}{\partial z}(M)dz = 2 \cdot e^5 dy + 4e^5 dz.$$

b) Write the function $u = f(x + y^2, y + x^2)$ in the form , where . $u = f(t, v) t = x + y^2, v = y + x^2$

By calculating partial derivatives and , we get .$\frac{\partial u}{\partial x} \frac{\partial u}{\partial y}$

$$\frac{\partial u}{\partial x} = f_t(x + y^2, y + x^2) \cdot 1 + f_v(x + y^2, y + x^2) \cdot 2x,$$

$$\frac{\partial u}{\partial x}(M) = f_t(0,2) - 2f_v(0,2),$$

$$\frac{\partial u}{\partial y} = f_t(x + y^2, y + x^2) \cdot 2y + f_v(x + y^2, y + x^2) \cdot 1,$$

$$\frac{\partial u}{\partial y}(M) = 2f_t(0,2) + f_v(0,2).$$

Therefore

$$du|_M = \frac{\partial u}{\partial x}(M)dx + \frac{\partial u}{\partial y}(M)dy = [f_t(0,2) - 2f_v(0,2)]dx + [2f_t(0,2) + f_v(0,2)]dy.$$

The same expression for can be obtained in another way, using the invariance of the form of the first differential. $du|_M$

Due to the invariance of the form of the first differential, we have

$$du|_M = = f_t(0,2)dt + f_v(0,2)dv,$$

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where and are the differentials of functions and at point . $dt\ dvt = x + y^2 v = y + x^2 M(-1;1)$
Since

$$\frac{\partial t}{\partial x}(M) = 1,,,, \frac{\partial t}{\partial y}(M) = 2\frac{\partial v}{\partial x}(M) = -2\frac{\partial v}{\partial y}(M) = 1$$

that

$$dt|_M = dx + 2dy, dv|_M = -2dx + dy$$

and we get

$$du|_M = f_t(0,2)(dx + 2dy) + f_v(0,2)(-2dx + dy) =$$
$$= [f_t(0,2) - 2f_v(0,2)] + [2f_t(0,2) + f_v(0,2)]dy,$$

which is the same as the equation given above.
Example 6. Prove that the

$$f(x) = f(x_1, x_2, ..., x_m) = x_1^2 + x_2^2 + ... + x_m^2$$

is differentiable at . $\forall (x_1^0, x_2^0, ..., x_m^0) \in R^m$

Find the full increment of the function at : $x^0 = (x_1^0, x_2^0, ..., x_m^0)$

$$\Delta f(x^0) = (x_1^0 + \Delta x_1)^2 + (x_2^0 + \Delta x_2)^2 + ... + (x_m^0 + \Delta x_m)^2 -$$
$$-(x_1^{0^2} + x_2^{0^2} + ... + x_m^{0^2}) = 2x_1^0 \Delta x_1 + 2x_2^0 \Delta x_2 + ... +$$
$$+2x_m^0 \Delta x_m + \Delta x_1^2 + \Delta x_2^2 + ... + \Delta x_m^2.$$

If

$$A_1 = 2x_1^0, A_2 = 2x_2^0, ..., A_m = 2x_m^0,$$
$$\alpha_1 = \Delta x_1, \alpha_2 = \Delta x_2, ..., \alpha_m = \Delta x_m$$

then

$$\Delta f(x^0) = A_1 \Delta x_1 + A_2 \Delta x_2 + ... + A_m \Delta x_m + \alpha_1 \Delta x_1 + \alpha_2 \Delta x_2 + ... + \alpha_m \Delta x_m \ .$$

From this it follows that the function differentiated at . $\forall x^0 \in R^m$

Example 7. Find Partial Derivatives of Functions

$$f(x, y) = \ln tg\frac{x}{y}.$$

Decision.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\ln tg\frac{x}{y}\right) = \frac{1}{tg\frac{x}{y}} \cdot \frac{1}{\cos^2\frac{x}{y}} \cdot \frac{1}{y} = \frac{2}{y\sin\frac{2x}{y}};$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\ln tg\frac{x}{y}\right) = \frac{1}{tg\frac{x}{y}} \cdot \frac{1}{\cos^2\frac{x}{y}} \cdot \left(-\frac{x}{y^2}\right) = \frac{-2}{y^2\sin\frac{2x}{y}}.$$

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Example 8. Compute Partial Derivatives of Functions

$$f(x,y)=\sqrt{x^2+y^2}.$$

Decision. Let $(x,y)\neq(0,0)$. Then

$$\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\sqrt{x^2+y^2}=\frac{2x}{2\sqrt{x^2+y^2}}=\frac{x}{\sqrt{x^2+y^2}},$$

$$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\sqrt{x^2+y^2}=\frac{2y}{2\sqrt{x^2+y^2}}=\frac{y}{\sqrt{x^2+y^2}}.$$

Let us now suppose that $(x,y)=(0,0)$. By definition

$$\frac{\partial f(0,0)}{\partial x}=\lim_{\Delta x\to 0}\frac{f(0+\Delta x,0)-f(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{|\Delta x|}{\Delta x},$$

$$\frac{\partial f(0,0)}{\partial y}=\lim_{\Delta y\to 0}\frac{f(0,0+\Delta y)-f(0,0)}{\Delta y}=\lim_{\Delta y\to 0}\frac{|\Delta y|}{\Delta y}.$$

It follows that the function has no partial derivatives at (0,0).

Example 8. Compute $\frac{\partial f}{\partial r}, \frac{\partial f}{\partial \varphi}$ if the function $f(x,y)$ is differentiable in R^2 and $x=r\cos\varphi$, $y=r\sin\varphi$

.

Decision. Since

$$f(x,y)=f(r\cos\varphi, r\sin\varphi).$$

By calculating the partial derivative rule of a complex function, we find:

$$\frac{\partial f}{\partial r}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r}=\cos\varphi\frac{\partial f}{\partial x}+\sin\varphi\frac{\partial f}{\partial y}=\frac{1}{\sqrt{x^2+y^2}}(x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}),$$

$$\frac{\partial f}{\partial \varphi}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \varphi}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \varphi}=-r\sin\varphi\frac{\partial f}{\partial x}+r\cos\varphi\frac{\partial f}{\partial y}=-y\frac{\partial f}{\partial x}+x\frac{\partial f}{\partial y}.$$

Example 9. Prove that the

$$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, & \text{если} \quad (x,y)\neq(0,0) \\ 0 \quad, & \text{если} \quad (x,y)=(0,0)\end{cases},$$

has partial derivatives at $(0,0)$, but is not differentiable at that point.

It's clear that,

$$\frac{\partial f(0,0)}{\partial x}=\lim_{\Delta x\to 0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x}=0,$$

$$\frac{\partial f(0,0)}{\partial y}=\lim_{\Delta y\to 0}\frac{f(0,\Delta y,)-f(0,0)}{\Delta y}=0,$$

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i.e. the function has partial derivatives at the point $(0,0)$ and is equal to zero.

Let's assume the opposite. Let the function be differentiable at (0,0):

$$\Delta f(0,0)=\frac{\partial f(0,0)}{\partial x}\Delta x+\frac{\partial f(0,0)}{\partial y}\Delta y+\alpha_1\Delta x+\alpha_2\Delta y=$$

$$=\alpha_1\Delta x+\alpha_2\Delta y\ . \qquad (\Delta x\to 0,\ \Delta y\to 0\ \partial a\ \ \alpha_1\to 0\ ,\ \alpha_2\to 0).$$

On the other hand

$$\Delta f(0,0)=f(0+\Delta x,\ 0+\Delta y)-f(0,0)=f(\Delta x,\Delta y)=\frac{\Delta x\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}\cdot$$

Mean

$$\frac{\Delta x\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}=\alpha_1\Delta x+\alpha_2\Delta y.$$

If $\Delta x=\Delta y>0$, then

$$\alpha_1+\alpha_2=\frac{1}{\sqrt{2}}\cdot$$

It follows that in this case $\Delta x=\Delta y>0$ the sum is not zero. And this contradicts that $\alpha_1\to 0,\alpha_2\to 0$
This means that the function is not differentiable at (0,0).

Methodology

Note that the theory of functions of many variables, especially the theory of partial differential and the differentiability of functions in dimensional space, is widely used in almost all areas of mathematics, mechanics, and biology. Examples of this are [2-14] research in these areas, based on advanced pedagogical technologies, and scientific articles [15-35], where the functions of many variables are investigated and applied in practical problems.m –

Conclusion

In conclusion, it should be noted that in the proposed methodology for teaching the calculation of partial derived functions of many variables, the main attention is paid to the presentation of educational materials from simple to complex. Here, special attention is paid to the composition of questions and tasks (questions and tasks fully covered this topic) for solving in a practical lesson and independent work, as well as active communication with students.

Independent study of the proposed questions and tasks develops students' skills in calculating partial derived functions of many variables. When compiling questions and tasks for independent work, student co-authors (who studied the topics in advance) made a large contribution. The scheme of teaching the topic proposed in the article (some methods for calculating partial derived functions of many variables) has been repeatedly positively assessed by students.

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REFERENCES

1. Butuzov V.P. Matematicheskiy analiz v voprosy i zadach [Mathematical Analysis in Questions and Problems], Moscow, Fizmatlit Publ., 2001, 479 p. (in Russian).
2. Rasulov Kh.R., Raupova M.Kh. Mathematical Models and Laws in Biology // Scientific progress, 2:2 (2021), p.870-879.
3. Rasulov Kh.R., Yashieva F.Yu. On Some Volterran Quadratic Stochastic Operators of a Bisexual Population with Continuous Time // Science, Technology and Education, 77:2-2 (2021), pp.23-26.
4. Rasulov Kh.R., Kamariddinova Sh.R. Ob odnogo dinamicheskoy sistema s nezprevnym vremya [On one dynamic system with continuous time] // "The XXI Century Skills for Professional Activity" International Scientific-Practical Conference, Tashkent, mart 2021 y., p.115-116.
5. Rasulov Kh.R., Raupova M.Kh., Yashieva F.Yu. Ikki Ginsley Population Wa Uning Mathematician Models // Science and Education, scientific journal, 2:10 (2021), p.81-96.
6. Rasulov Kh.R., Yashieva F.Y. Ob odnom quadraticheskogo stochasticheskogo operator s neprezvlennym vremya [On one quadratic stochastic operator with continuous time] // "The XXI Century Skills for Professional Activity" International Scientific-Practical Conference, Tashkent, mart 2021 y., p.145-146.
7. Расулов Х.Р., Яшиева Ф.Ю. Икки жинсли популяциянинг динамикаси ҳақида // Scientific progress, 2:1 (2021), p.665-672.
8. Rasulov Kh.R., Kamariddinova Sh.R. On the Analysis of Some Non-Volterran Dynamic Systems with Continuous Time // Science, Technology and Education, 77:2-2 (2021) pp.27-30.
9. Rasulov X.R., Qamariddinova Sh.R. On the analysis of some dynamic systems / Scientific progress, 2:1 (2021), pp. 448-454.
10. Rasulov Kh.R., Dzhurakulova F.M. On a Dynamic System with Non-Continuous Time // Science, Technology and Education, 77:2-2 (2021), p. 19-22.
11. Х.Р Расулов, Ф.М. Джуракулова (2021). Баъзи динамик системаларнинг сонли ечимлари ҳақида // Scientific progress. 2:1, 455-462 бетлар.
12. Rasulov Kh.R., Raupova M.Kh. The Role of Mathematics in Biological Sciences // Problems of Pedagogy, no. 53:2 (2021), pp. 7-10.
13. Расулов Х.Р., Собиров С.Ж. Модуль қатнашган баъзи тенглама, тенгсизлик ва тенгламалар системаларини ечиш йўллари // Science and Education, scientific journal, 2:9 (2021), p.7-20.
14. Расулов Х.Р., Собиров С.Ж. Айрим рационал тенгламаларни ечишда интерфаол усулларни қўлланилиши ҳақида // Science and Education, scientific journal, 2:10 (2021), p. 586-595.
15. Расулов Х.Р., Собиров С.Ж. Айрим иррационал тенгламаларни ечишда интерфаол усулларни қўлланилиши // Science and Education, scientific journal, 2:10 (2021), p.596-607.
16. Rasulov X.R., Sayfullayeva Sh.Sh. (2022) About the boundary issue for a quasi-elliptical equation with two breakdown lines / Central Asian Academic Journal Of Scientific Research, 2(5), 544-557 b.
17. Rasulov X.R. Sayfullayeva Sh.Sh. About boundary issues for elliptical-type equations with a breakdown line / Science and Education, scientific journal, 3:3 (2022), pp. 46-54.

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18. Rasulov Kh.R. O nekotorykh simvolakh matematicheskogo analiza [On some symbols of mathematical analysis] // Science and Education, scientific journal, 2:11 (2021), p.66-77.
19. Rasulov Kh.R. On the Concept of Asymptotic Decomposition and Its Some Applications // Science and Education, scientific journal, 2:11 (2021), pp.77-88.
20. Xaydar R. Rasulov. On the solvability of a boundary value problem for a quasilinear equation of mixed type with two degeneration lines // Journal of Physics: Conference Series 2070 012002 (2021), pp.1–11.
21. Rasulov H. KD problem for a quasilinear equation of an elliptic type with two lines of degeneration // Journal of Global Research in Mathematical Archives. 6:10 (2019), p.35-38.
22. Rasulov Kh.R., Sobirov S.Zh. A problem of the type of Gellerstedt problems for a single equation of mixed type with two lines of degeneracy // Scientific progress, 2:1 (2021), p.42-48.
23. Salokhitdinov, M.S., Rasulov, Kh.R. (1996). Cauchy's Problem for a Quasilinear Degenerate Equation of a Hyperbolic Type // DAN of the Republic of Uzbekistan, No. 4, pp. 3-7.
24. Rasulov X.R. Boundary value problem in a domain with deviation from the characteristics for one nonlinear equation of a mixed type // Modern problems of applied mathematics and information technologies Al-Khwarizmi 2021, Fergana, Uzbekistan, p. 149.
25. Rasulov X.R. (2020). Boundary value problem for a quasilinear elliptic equation with two perpendicular line of degeneration // Uzbek Mathematical Journal, №3, pp.117-125.
26. Rasulov, Kh.R. (1996). The Dirichlet Problem for a Quasilinear Equation of an Elliptic Type with Two Degeneracy Lines // DAN of the Republic of Uzbekistan, No. 12, pp. 12-16.
27. Rasulov Kh.R. (2018). On a continuous time F - quadratic dynamical system // Uzbek Mathematical Journal, №4, pp.126-131.
28. Rasulov H. Boundary value problem for a quasilinear elliptic equation with two perpendicular line of degeneration // Центр научных публикаций (buxdu. uz) 5:5 (2021).
29. Rasulov Kh.R., Sobirov S.Zh. A problem of the Gellerstedt type for one mixed-type equation with two lines of degeneration // Scientific progress, 2:1 (2021), p. 42-48.
30. Islomov B., Rasulov Kh.R. (1997). Existence of generalized solutions of the boundary value problem for a quasilinear equation of mixed type with two lines of degeneracy // DAN of the Republic of Uzbekistan, No. 7, pp. 5-9.
2318. Rasulov Kh.R. et al. On the solvability of the Cauchy problem for a degenerate quasilinear equation of a hyperbolic type. 53:6-1, 2019. P.16-18.
32. Rasulov Kh.R. Ob odnoj nelocal'noy zadach dlya raveniya hyperbolicheskogo tipa [On one nonlocal problem for the equation of hyperbolic type] // XXX Krymskaya Osennyaya Matematicheskaya Shkola-

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symposium po spektral'nym i evolutsionnym zadach [On one non-local problem for the equation of hyperbolic type]. Proceedings of the International Conference KROMSH-2019, pp. 197-199.
33. Rasulov Kh.R., Rashidov A.Sh. On the existence of a generalized solution of the boundary problem for a nonlinear equation of a mixed type // Bulletin of Science and Education, 97:19-1 (2020), pp. 6-9.
34. Rasulov, H. (2022). Boundary value problems for mixed-type quasilinear equations with two lines of degeneracy. Centre for Scientific Publications (buxdu. Uz), 8(8).
35. Rasulov Kh.R., Sayfullayeva Sh.Sh. Analysis of Some Boundary Value Problems for Mixed-Type Equations with Two Lines of Degeneracy // Irish Interdisciplinary Journal of Science & Research (IIJSR), 6:2 (2022), p. 8-14.