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MATHEMATICAL MODELING OF ECONOMIC GROWTH IN UZBEKISTAN: A CALIBRATED AUGMENTED SOLOW MODEL WITH PHYSICAL AND HUMAN CAPITAL

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Abstract
This paper develops and numerically solves a dynamic mathematical model of economic growth for Uzbekistan based on the augmented Solow framework with physical and human capital. The model is formulated as a system of two nonlinear differential equations for capital per effective worker, with a Cobb–Douglas production function. Parameters are calibrated to the structural characteristics of the Uzbek economy: a high investment rate, a young and growing population, and an expanding education sector. The steady state is derived analytically, the speed of convergence is obtained by linearization, and transitional dynamics are simulated with a fine-step explicit Euler scheme over a 30-year horizon. Four policy scenarios are compared. The results show that the convergence rate is about 3.6% per year, implying a half-life of roughly 19 years. Raising the investment rate from 35% to 40% of GDP increases steady-state output per effective worker by 12.4%, while raising human capital investment from 6% to 8% of GDP increases it by 19.7%. A combined policy yields a 34.5% gain. The findings suggest that, at Uzbekistan's current investment intensity, further returns from physical capital accumulation are smaller than those from human capital, and that education and skills policy should be treated as a core element of the national growth strategy.

Keywords: mathematical modeling; economic growth; augmented Solow model; Cobb–Douglas production function; human capital; differential equations; convergence; Uzbekistan.

1. Introduction

Mathematical models are the main tool for turning growth theory into quantitative guidance for economic policy. For a transition economy such as Uzbekistan, which has pursued large-scale market reforms since 2017, the question is no longer only how fast the economy grows, but which sources of growth will sustain it over the coming decades.

Uzbekistan combines three features that make a formal growth model especially useful. First, gross capital formation has remained high, at roughly a third or more of GDP in recent years. Second, the population is young and grows at about 2% per year, so a large share of investment is needed simply to equip new workers. Third, the government has expanded higher education and vocational training, raising the question of how human capital interacts with physical capital.

The theoretical foundation of this study is the neoclassical growth model of Solow (1956) and Swan (1956), in which output is produced with capital and labor under diminishing returns. Mankiw, Romer and Weil (1992) augmented the model with human capital and showed that it explains a large part of

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cross-country income differences. Endogenous growth theory (Lucas, 1988; Romer, 1990) later emphasized knowledge and skills as drivers of long-run growth. Empirical work on convergence (Barro and Sala-i-Martin, 2004; Islam, 1995) and development accounting (Hall and Jones, 1999; Caselli, 2005) has confirmed that both physical and human capital matter, while Hanushek and Woessmann (2008) stressed the quality of skills rather than years of schooling alone.

Most applications of these models concern advanced economies or large cross-country panels. Few studies provide a transparent, calibrated dynamic model for Uzbekistan that derives the steady state, the speed of convergence and the effects of alternative investment policies within one consistent framework. This paper addresses that gap.

The aim of the study is to build a calibrated augmented Solow model for the Uzbek economy and to use it to answer two questions: (1) how fast the economy converges to its long-run path; and (2) whether a marginal increase in investment yields larger gains when directed to physical capital or to human capital.

2. Methods

2.1. Model specification

The economy produces a single good with physical capital K, human capital H and labor augmented by technology AL, using a Cobb–Douglas production function:

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} [A(t)L(t)]^{1-\alpha-\beta}, \qquad \alpha > 0,\ \beta > 0,\ \alpha + \beta < 1 \qquad (1)$$

Labor and technology grow at constant rates n and g. A share s_k of output is invested in physical capital and a share s_h in human capital; both depreciate at rate δ. In units per effective worker, $k = K/(AL)$, $h = H/(AL)$, $y = Y/(AL)$, the model reduces to a system of two nonlinear differential equations:

$$\dot{k}(t) = s_k\, k(t)^{\alpha} h(t)^{\beta} - (n + g + \delta)\, k(t) \qquad (2)$$

$$\dot{h}(t) = s_h\, k(t)^{\alpha} h(t)^{\beta} - (n + g + \delta)\, h(t) \qquad (3)$$

2.2. Steady state and convergence

Setting $\dot{k} = \dot{h} = 0$ gives the unique positive steady state:

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n + g + \delta} \right)^{\frac{1}{1-\alpha-\beta}} \qquad (4)$$

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n + g + \delta} \right)^{\frac{1}{1-\alpha-\beta}} \qquad (5)$$

Steady-state output per effective worker is

$$y^* = (k^*)^{\alpha} (h^*)^{\beta} \qquad (6)$$

Linearizing system (2)–(3) around the steady state yields the speed of convergence λ and the half-life $t_{1/2}$ of the gap between current and steady-state output:

$$\lambda = (1 - \alpha - \beta)(n + g + \delta), \qquad t_{1/2} = \frac{\ln 2}{\lambda} \qquad (7)$$

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2.3. Calibration

Parameters are set to reflect the structure of the Uzbek economy and standard values from the growth literature (Table 1). The investment rate approximates the recent ratio of gross capital formation to GDP; population growth approximates recent demographic data; the human capital share follows Mankiw, Romer and Weil (1992). The values are approximate and serve as a benchmark rather than precise point estimates.

Table 1. Benchmark calibration

Parameter	Symbol	Value	Basis
Physical capital elasticity	α	0.35	Capital share in national income, standard range 0.30–0.40
Human capital elasticity	β	0.25	Mankiw, Romer and Weil (1992)
Investment rate, physical capital	s_k	0.35	Approximate gross capital formation / GDP, Uzbekistan
Investment rate, human capital	s_h	0.06	Approximate spending on education and training / GDP
Population (labor) growth	n	0.02	Approximate recent population growth
Technology growth	g	0.02	Standard assumption
Depreciation rate	δ	0.05	Standard assumption

2.4. Simulation design

The system is solved numerically with an explicit Euler scheme with step $\Delta t = 0.01$ years over a 30-year horizon:

$$k_{t+\Delta t} = k_t + \Delta t\left[s_k k_t^{\alpha} h_t^{\beta} - (n + g + \delta)k_t\right], \qquad h_{t+\Delta t} = h_t + \Delta t\left[s_h k_t^{\alpha} h_t^{\beta} - (n + g + \delta)h_t\right] \qquad (8)$$

The economy starts with k and h at 60% of their baseline steady-state values, reflecting a catching-up economy below its long-run path. Four scenarios are compared: S0 (baseline: $s_k = 0.35$, $s_h = 0.06$); S1 (higher physical investment: $s_k = 0.40$); S2 (higher human capital investment: $s_h = 0.08$); and S3 (both increases combined). All output values are expressed relative to baseline steady-state output ($y^*_{S0} = 1$).

3. Results

3.1. Steady state and speed of convergence

Under the benchmark calibration the convergence rate is

$$\lambda = (1 - 0.35 - 0.25)(0.02 + 0.02 + 0.05) = 0.036$$

or 3.6% per year. The half-life is $t_{1/2} = \ln 2/0.036 \approx 19.3$ years: an economy that starts below its long-run path closes half of the gap in about two decades. The baseline steady-state capital–output ratio is 3.89.

Table 2 reports the long-run effects of the four scenarios. Higher human capital investment (S2) raises steady-state output more than higher physical investment (S1), although S1 adds 5 percentage points of GDP to investment and S2 adds only 2.

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Table 2. Steady-state values by scenario (per effective worker)

Scenario	s_k	s_h	k^*	h^*	y^*	y^*/y^*_{S0}	K/Y
S0 Baseline	0.35	0.06	9.905	1.698	2.547	1.000	3.89
S1 Physical capital	0.40	0.06	12.723	1.908	2.863	1.124	4.44
S2 Human capital	0.35	0.08	11.856	2.710	3.049	1.197	3.89
S3 Combined	0.40	0.08	15.230	3.046	3.427	1.345	4.44

The long-run elasticities of output follow directly from the steady-state solution:

$$\frac{\partial \ln y^*}{\partial \ln s_k} = \frac{\alpha}{1-\alpha-\beta} = 0.875, \qquad \frac{\partial \ln y^*}{\partial \ln s_h} = \frac{\beta}{1-\alpha-\beta} = 0.625 \qquad (9)$$

Although the elasticity with respect to s_k is larger, the base level of s_h is much lower, so each additional percentage point of GDP invested in human capital raises output more.

3.2. Transitional dynamics

Figure 1 shows the simulated paths when the economy starts at 60% of its baseline steady-state capital stocks.

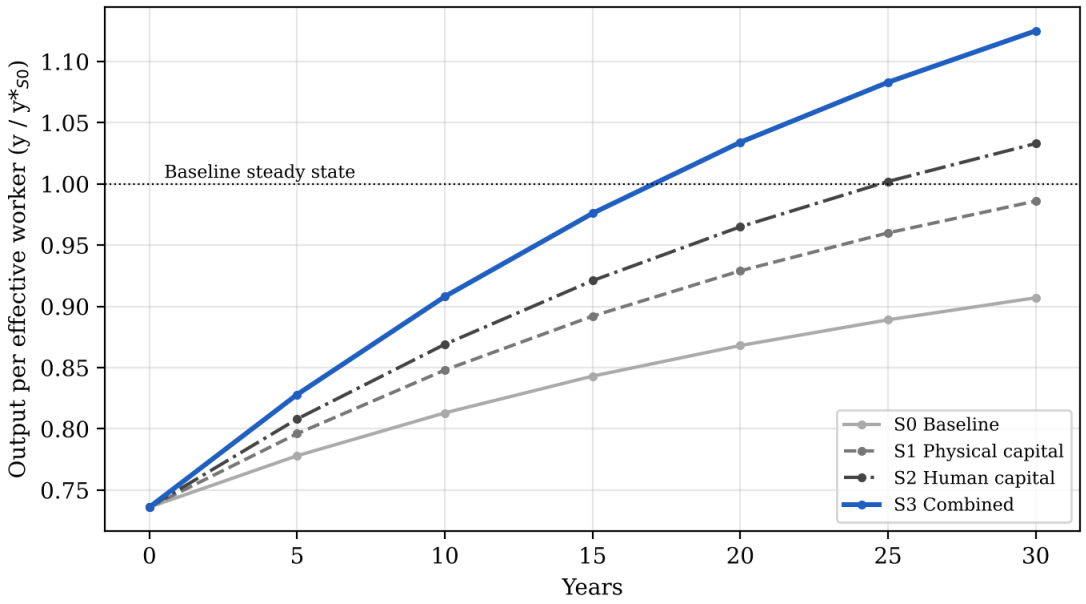


Figure 1. Transitional dynamics of output per effective worker, scenarios S0–S3 (author's simulation of the calibrated model).

After 30 years the baseline economy reaches 0.907 of its steady state, consistent with a half-life of about 19 years. Only the scenarios with higher human capital investment cross the baseline steady-state level within the horizon: S2 after about 25 years and S3 after about 17 years. The gap between S3 and S0 widens over time, from 11.7% after 10 years to 24.0% after 30 years.

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4. Discussion

The results lead to three main conclusions about growth in Uzbekistan.

First, the estimated convergence rate of 3.6% per year is close to the 2–3% range commonly reported in cross-country convergence studies (Barro and Sala-i-Martin, 2004), and slightly higher because of the economy's rapid population growth. In practice this means that structural reforms affect living standards gradually: about half of their long-run effect appears within two decades. Policy evaluation should therefore use a long horizon rather than annual growth figures alone.

Second, the model indicates diminishing returns to further physical capital accumulation. With an investment rate already near 35% of GDP, raising it to 40% increases the steady-state capital–output ratio from 3.89 to 4.44 but adds only 12.4% to long-run output. Each additional percentage point of GDP invested in physical capital raises steady-state output by about 2.5%, whereas each additional point invested in human capital raises it by about 9.9%. This asymmetry follows from the relatively low starting level of human capital investment, not from a larger elasticity.

Third, physical and human capital are complements. The combined scenario S3 yields a 34.5% gain, which is larger than the sum of the separate gains (12.4% + 19.7% = 32.1%). Equipment and infrastructure are used more productively by a skilled labor force, and skilled workers earn higher returns when they have more capital to work with.

These findings have direct policy implications. Investment programs should be accompanied by measures that raise the quality and coverage of education, vocational training and research. Priorities include closer links between universities and employers, expanded training in engineering, digital and managerial skills, and incentives for firms to invest in on-the-job training. Public investment appraisal could include human capital effects as an explicit criterion.

The study has several limitations. The calibrated parameters are approximate; precise estimates of α and β for Uzbekistan would require firm-level or national accounts data on factor shares. The model treats technology growth as exogenous and ignores the quality of institutions, foreign trade, remittances and the sectoral structure of the economy. Human capital is measured by investment flows, while the quality of skills (Hanushek and Woessmann, 2008) may matter more. Future research could estimate the model econometrically on regional panel data and extend it to an endogenous growth framework.

5. Conclusion

This paper built a calibrated augmented Solow model of the Uzbek economy, derived its steady state and speed of convergence analytically, and simulated four investment scenarios over 30 years. The economy converges to its long-run path at about 3.6% per year, with a half-life of roughly 19 years. Given the already high investment rate, an additional percentage point of GDP directed to human capital raises long-run output about four times more than the same point directed to physical capital. A combined policy produces the largest gain, 34.5%, because the two forms of capital are complementary.

The main practical conclusion is that sustained growth in Uzbekistan will depend increasingly on the quality of its workforce. Mathematical modeling of this kind provides a transparent tool for comparing

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policy options and can be extended with econometric estimation and endogenous technology in further research.

References

1. Barro, R. J., and Sala-i-Martin, X. (2004). Economic Growth (2nd ed.). Cambridge, MA: MIT Press.
2. Caselli, F. (2005). Accounting for cross-country income differences. In P. Aghion and S. Durlauf (Eds.), Handbook of Economic Growth (Vol. 1A, pp. 679–741). Amsterdam: Elsevier.
3. Cobb, C. W., and Douglas, P. H. (1928). A theory of production. American Economic Review, 18(1), 139–165.
4. Hall, R. E., and Jones, C. I. (1999). Why do some countries produce so much more output per worker than others? Quarterly Journal of Economics, 114(1), 83–116.
5. Hanushek, E. A., and Woessmann, L. (2008). The role of cognitive skills in economic development. Journal of Economic Literature, 46(3), 607–668.
6. Islam, N. (1995). Growth empirics: A panel data approach. Quarterly Journal of Economics, 110(4), 1127–1170.
7. Lucas, R. E. (1988). On the mechanics of economic development. Journal of Monetary Economics, 22(1), 3–42.
8. Mankiw, N. G., Romer, D., and Weil, D. N. (1992). A contribution to the empirics of economic growth. Quarterly Journal of Economics, 107(2), 407–437.
9. Romer, P. M. (1990). Endogenous technological change. Journal of Political Economy, 98(5), S71–S102.
10. Solow, R. M. (1956). A contribution to the theory of economic growth. Quarterly Journal of Economics, 70(1), 65–94.
11. Swan, T. W. (1956). Economic growth and capital accumulation. Economic Record, 32(2), 334–361.
12. World Bank. World Development Indicators. Washington, DC: The World Bank. https://databank.worldbank.org/source/world-development-indicators